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On the optimum intensity response in the quasi-optical millimetre wave band for a transmission echelon grating

Sonali Chakrabarti*
International Centre for Theoretical Physics, Strada Costiera, 34100 Trieste, Italy

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Abstract : A thorough theoretical analysis was made for the optimisation of intensity for a transmission echelon grating in the quasi-optical millimetre wave region It appeared that the geometrical dimensions of the grating play a crucial role for obtaining maximum intensity for a particular wavelength. The problem of proper choice of geometrical parameters has been tackled from contour plots of 3D-intensity curves. The method affords a practical possibility for obtaining maximum intensity with minimum geometrical dimension of the grating This will thus, reduce the construction cost of such echelons which gives an additional advantage apart from simultaneously producing the highest level of intensity at the desired wavelength.

Keywords : Millimetre wave diffraction, intensity response, echelon transmission grating, quasi-optical waves

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1. Introduction

In our previous communication [1], we have examined the diffraction pattern produced by an echelon transmission grating both for optical and quasi-optical millimetre waves. An expression for the diffracted intensity, has also been derived. Echelon transmission grating, as we all know, is entirely different from a plane grating in its construction. It was first invented by Michelson [2], who arranged a series of parallel glass plates in the form of a flight of steps. With such an echelon, one can observe the spectrum of a very high order and hence in the optical region, the resolving power of the instrument becomes very high. It is

*Department of Physics, Jadavpur University, Calcutta-700 032, India

therefore, very useful in detecting the true monochromatism of a beam of light and also studying the hyperfine structure *e.g.* the splitting of spectral lines in Zeeman effect. In the last two or three decades, there has been a remarkable improvement in the grating technology [3-10]. With the advent of lasc , holography and Fourier transform spectroscopy using lenses and gratings, there is considerable development in studying the strains [11], photoelasticity [12,13] and topography [14,15] of intricate objects. The method also provides new possibilities for investigating pulsed and stationary phase heterogenetics—gas streams, flames, explosions, shock waves and plasma [16-23].

In recent times, the microwaves and millimetre waves are continuously gaining importance in communication link and remote sensing [24,25]. The pioneering work with millimetre wave grating began as early as 1897 by Sir J. C. Bose who at first determined the electronic radiation by a diffraction grating [26]. He designed microwave spectrometer, diffraction gratings, polarimeters, spark generators and coherer detectors and successfully conducted several microwave experiments at 5 mm [27, 28].

These microwaves are endowed with several important characteristics. They are neither reflected nor absorbed by the atmosphere. In fact their absorption is too small to be accounted for. This is why microwaves are widely used now a days, by radio-astronomers to study electromagnetic waves originating from stars and other astronomical objects. Microwave holography revolutionised the investigation process for exploring the surface of earth, moon and planets from satellites [29-33]. The principles of zone plates and gratings are also being utilised in constructing microwave antennas and wave guides [34–38].

In the light of the above discussion, it seems that Echelon transmission grating, if used in the millimetre and microwave band might have prospective scope in diverse fields specially in radio-astronomy It was therefore, considered necessary to study the microwave diffraction by such echelons elaborately. As already pointed out, we have derived an expression for its intensity as well as its resolving power. It was found that the geometrical dimension play an important role in getting maximum intensity for a particular wavelength [1]. In the present paper we have analysed theoretically, how the proper choice of geometrical parameters be made so as to obtain highest diffracted intensity with its minimum dimension. This will make the echelon occupy the smallest possible space so that it becomes handy and at the same time, keep its construction cost minimum without sacrificing its highest level of intensity. This is of great significance in the investigation with millimetre and microwaves by a quasi-optical transmission grating specially in astrophysics and space research where one is interested in concentrating the most of the electromagnetic energy in a specified direction.

2. Millimetre wave echelon transmission grating

For use of echelon transmission grating in the quasi-optical millimetre wave region, the construction of ordinary echelon grating requires modification. In this case, Mylar may be chosen as suitable grating material which transmits millimetre waves with negligible absorption. The breadths and depths should be of the order of a centimetre. The transmission echelon grating of such type has been represented in Figure 1. Let N be the number of steps, b be the breadth and d the depth of each step, μ the refractive index of the material

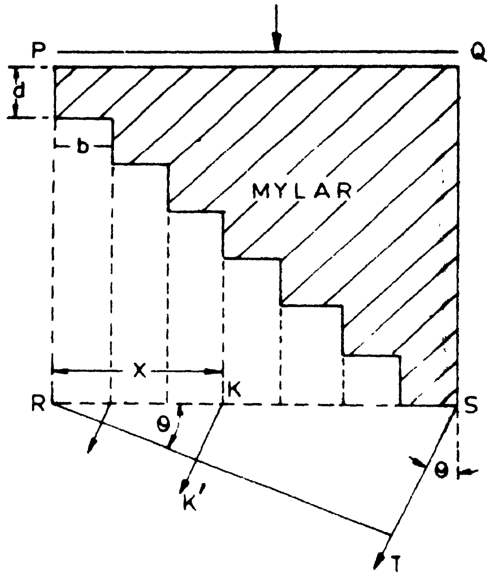

Figure 1. Ray diagram of the millimetre wave echelon transmission grating is shown with principal geometrical parameters. Here PQ = Incident wave front and RT = Diffracted wave front

corresponding to the wavelength λ and θ the angle of diffraction. The intensity of the diffracted wave is given by [1]

$$J = \left\{ \frac{b \sin \dfrac{\pi b \sin\theta}{\lambda}}{\dfrac{\pi b \sin\theta}{\lambda}} \right\}^2 \times \left\{ \frac{\dfrac{\sin \pi N(\mu - 1)d + b \sin\theta}{\lambda}}{\dfrac{\sin \pi(\mu - 1)d + b \sin\theta}{\lambda}} \right\}^2 \tag{1}$$

The resolving power of such a grating is given by,

$$\frac{\lambda}{d\lambda} = \frac{Nd \left[(\mu - 1) - \lambda \dfrac{d\mu}{d\lambda} \right]}{\lambda} \tag{2}$$

Neglecting $\dfrac{d\mu}{d\lambda}$, we arrive at an approximate for its resolving power as

$$\frac{\lambda}{d\lambda} = \frac{Nd\ (\mu - 1)}{\lambda} \tag{3}$$

It can easily be shown that for the principal maxima,

$$(\mu - 1)\, d + b\theta = n\lambda, \tag{4}$$

where n denotes the order of the spectra.

Neglecting $b\,\theta$, we obtain

$$(\mu - 1)\,d = n\lambda$$

or,
$$\frac{\lambda}{d\lambda} = \frac{(\mu - 1)\,d}{\lambda} = Nn\,. \tag{5}$$

Thus, the resolving power of a transmission echelon is approximately given by the product of the number of steps of the echelon and the order of the spectra.

Such echelons may be illuminated by microwaves or millimetre waves obtained from a Gunn diode or Impatt diode or a semiconductor laser and the radiation is allowed to fall almost normally. The advantage of normal incidence of the radiation lies in the fact that the absorption by the grating material, though negligibly small, will be still reduced to minimum, thus increasing the diffraction intensity and efficiency.

3. Results and discussion

Let us consider the intensity of the diffracted waves from an echelon transmission grating as shown in Figure 2. Here the intensity has been plotted against wavelength with parameters $N = 30$, $b = d = 1$ cm,. $\mu = 1.5$ and $\theta = 5°$. The figure exhibits the existence of several

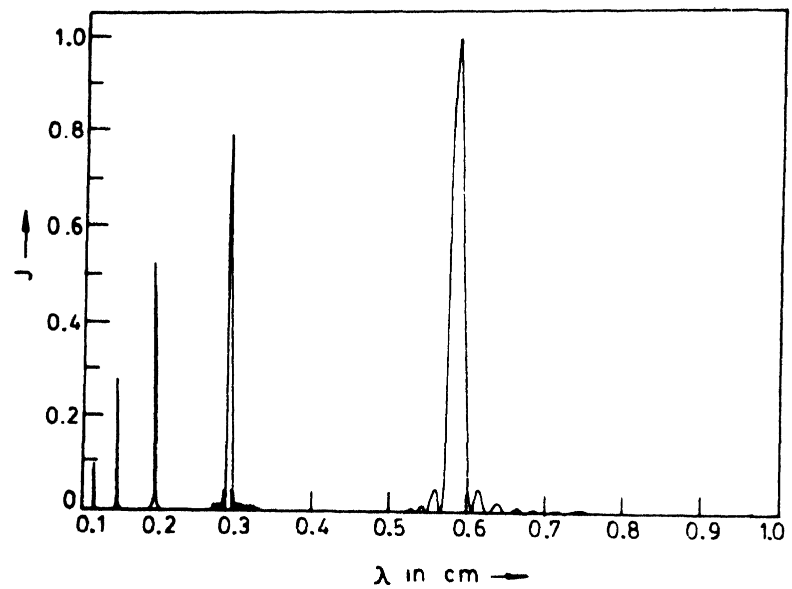

Figure 2. Intensity distribution as a function of the wavelength in the millimetre wave region The parameters are $N = 30$, $b = d = 1$ cm, $\mu = 1.5$, $\theta = 5°$.

peaks in the millimetre wave region in the vicinity of 1.2 mm, 1.5 mm, 2 mm, 3 mm and 5.9 mm the maximum peak being at 5.9 mm. If we reduce the number of steps say $N = 10$, keeping the other parameters fixed the basic nature of the intensity pattern remains almost unaltered, the maximum peak occurring again near 5.9 mm. The figure has not been shown here to avoid repetition. Although the resolving power of the instrument diminishes to one third when $N = 10$, the change in intensity is however, insignificant.

It may be mentioned in this connection that the grating efficiency in the case of microwave lamellar grating [39] was found to be as high as 95% if the number of corrugations is about 10. In the case of microwave transmission echelon also, it appears that the efficiency will remain quite high even if the number of steps is reduced to $N = 10$. Hence we can obtain good intensity with reasonably good efficiency if we choose $N = 10$, which will bring down the construction cost substantially. The reduction of the number of steps has thus two fold advantages, viz. economy of space and its construction cost.

A three dimensional plot for intensity distribution as a function of relative sizes of the steps *i.e.* b and d is represented in Figure 3. The parameters chosen are $N = 30$, $b = d = 1$ cm to 10 cm, $\lambda = 5$ mm and $\theta = 5°$. Another three dimensional plot for intensity distribution was obtained when the number of steps was made $N = 10$. This is represented in Figure 4.

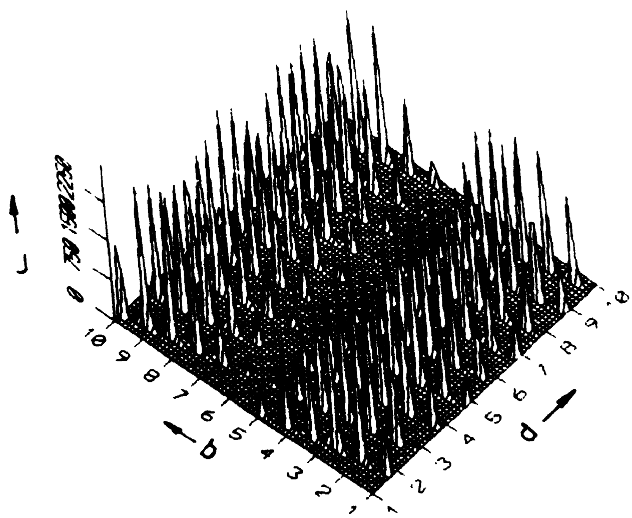

Figure 3. 3D plot for intensity distribution as a function of step parameters b and d. Here $N = 30$, $\mu = 1.5$, $\lambda = 0.5$ cm, b and d is varied from 1 cm to 10 cm

In order to draw a meaningful inference, let us confine our attention to the contour plot of Figure 4 which is shown in Figure 5 where $N = 10$. The minimum value of b and d which corresponds to the optimum intensity response are $b = 2.87$ cm and $d = 1.5$ cm for $\lambda = 0.5$ cm. The minimum geometrical parameter is selected because it will reduce its size and hence the construction cost. This is also very important from the economic point of view. For different wavelengths between 0.1 cm to 1.0 cm, the optimum response for intensity (J) at different b and d combination has been computed from the contour plot and presented in Table 1.

Table 1. Optimum intensity response with wavelength at various breadth and depth of the Echelon transmission grating.

Serial No.	Wavelength in cm	Optimum response for J at (b, d) in cm	
1	0.1	(1.72,	1.10)
2	0.2	(1.15,	1.40)
3	0.3	(1.72,	1.50)
4	0.4	(2.29,	1.20)
5	0.5	(2.87,	1.50)
6	0.6	(3.44,	1.80)
7	0.7	(4.02,	2.10)
8	0.8	(4.59,	2.40)
9	0.9	(5.17,	2.70)
10	1.0	(5 74,	3.0)

The above table indicates that the optimum response for J occurs when the values of breadth lies between $b = 1.15$ cm to $b = 5.74$ cm as the wavelength λ varies between 0.1 cm to 1.0 cm. On the other hand, for the same wavelength range the values of depth lies between

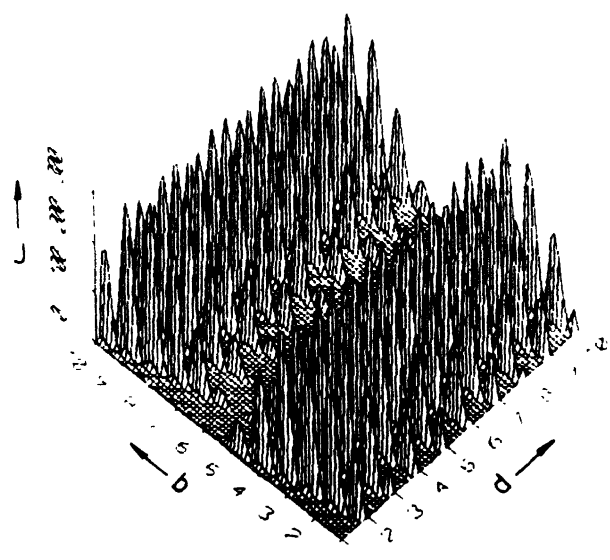

Figure 4. 3D plot for intensity distribution as a function of step parameters b and d. Here $N = 10$, $\mu = 1.5$, $\lambda = 0.5$ cm, b and d is varied from 1cm to 10 cm.

1.10 cm to 3.0 cm for optimum intensity. It therefore, appears that the optimum response is more sensitive on the values of breadth b, than on the values of depth d. It was further noticed that in order to get maximum intensity response for different values of wavelength, the geometrical parameters are to be made different. We wanted to examine how the step parameters are to be varied to get the maximum response within the wavelength range 0.1 cm to 1.0 cm. In order to make the echelon portable and economical, and at the same time diffract wavelengths upto 1 cm, the minimum b and d combination greater than 1.0 cm is taken as the best choice.

To draw the table, the contour plot was necessary to obtain the exact location for maximum J. The contour plot for λ = 0.5 cm, is shown in Figure 5, in which the best

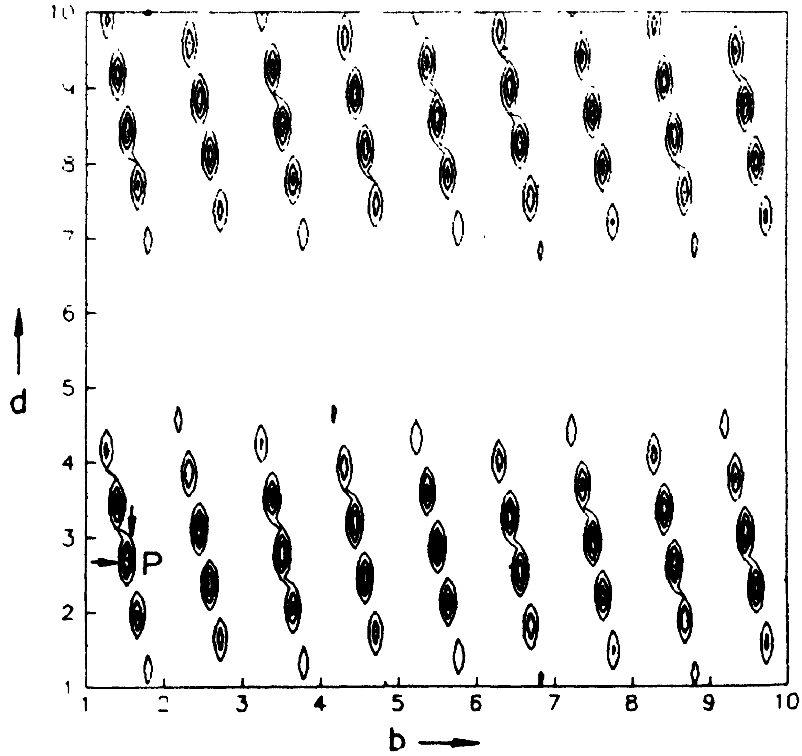

Figure 5. Contour plot of Figure 4. Here N = 10, μ = 1.5, λ = 0.5 cm, b and d is varied from 1 cm to 10 cm. The coordinates of P are b = 2.87 cm and d = 1.5 cm.

combination is b = 2.87 cm and d = 1.50 cm. Optimum (b , d) for other wavelengths are similarly obtained from the contour plots for those wavelengths, although these plots are not shown here to avoid repetition. Hence we can conclude that, the selection of geometrical parameters is extremely essential for optimum intensity response and such an ideal choice can be obtained from the contour plots of 3D-intensity curves. It would perhaps be not out of place to mention that we have analysed the diffraction intensity for echelon reflection grating [40] and the study of contour plots for such echelons appear necessary to ascertain the best step parameters in this case also.

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